A NOTE ON $k - \varepsilon$ MODELLING OF VEGETATION CANOPY AIR-FLOWS

Research Note

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Abstract. The $k-\varepsilon$ turbulence model is a standard of computational software packages for engineering, yet its application to canopy turbulence has not received comparable attention. This is probably due to the additional source (and/or sink) terms, whose parameterization remained uncertain. This model must include source terms for both turbulent kinetic energy (k) and the viscous dissipation rate (ε), to account for vegetation wake turbulence budget. In this note, we show how Kolmogorov's relation allows for an analytical solution to be calculated within the portion of a dense and homogeneous canopy where the mixing length does not vary. By substitution within model equations, this solution allows for a set of constraints on source term model coefficients to be derived. Those constraints should meet both Reynolds averaged Navier–Stokes equations and large-eddy simulation sub-grid scale turbulence modelling requirements. Although originating from within a limited portion of the canopy, the predicted coefficients values must be valid elsewhere in order to make the model capable of predicting the whole canopy-layer flow with a single set of constants.

Keywords: Turbid medium, Turbulence model, Turbulent kinetic energy, Vegetation canopy, Wakes.

1. Introduction

Two-equation turbulence models, such as the $k - \varepsilon$ one, are derived from the so called 1.5-closure model, which shares the assumption of an isotropic diffusivity with first-order closure models. In fact, both models collapse (Detering and Etling, 1985) for the inertial-sublayer similarity. However, two-equation models do not suffer from the same limitations as first-order turbulence models. The 1.5-closure model for kinematic turbulent viscosity (ν_t) can be matched (Lee, 1996) with roughness sublayer measurements, while two-equation models have a multi-dimensional modelling ability (Claussen, 1988). For these reasons, the $k - \varepsilon$ turbulence model is available within most software packages for turbulent flow computations. In addition, the $k - \varepsilon$ model can be used for large-eddy simulation (LES) sub-grid scale turbulence budget modelling (Kanda and Hino, 1994). The abilities of this model are due to the fact that ν_t is calculated from two flow properties: k and ε, for the $k - \varepsilon$ model. Because of their direct influence upon ν_t, the modelled budget equations for k and ε must account for vegetation wakes. This was modelled by Green (1992) through additional source terms that have a significant

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influence, even downwind of the canopy (Liu et al., 1996). Source terms for k and ε involve between three to four additional budget terms, whose dimensionless coefficients lack a firm physical basis in some cases. This may explain the limited use of $k - \varepsilon$ models in micrometeorology, despite their potential.

2. Model Equations

2.1. The $k - \varepsilon$ Model

As a starting point, we consider a one-dimensional steady state, neutrally stratified, fully developed surface boundary-layer flow within a dense and extensive planar homogeneous canopy. Within the canopy, the modelled budget equation for the mean averaged flow velocity (U) is

$$0 = \frac{d}{dz}\left(\nu_t \frac{dU}{dz}\right) + S_U, \tag{1}$$

where

$$\nu_t = C_\mu^{\frac{1}{4}} l_m k^{\frac{1}{2}}, \tag{2}$$

and l_m is the mixing length, and C_μ may depart from its standard engineering ($C_\mu = 0.09$) value for application to the atmospheric surface layer (Detering and Etling, 1985). The modelled budget equations for k

$$0 = \frac{d}{dz}\left(\frac{\nu_t}{\sigma_k}\frac{dk}{dz}\right) + \nu_t\left(\frac{dU}{dz}\right)^2 - \varepsilon + S_k \tag{3}$$

and ε

$$0 = \frac{d}{dz}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{d\varepsilon}{dz}\right) + C_{\varepsilon 1} C_\mu k \left(\frac{dU}{dz}\right)^2 - C_{\varepsilon 2}\frac{\varepsilon^2}{k} + S_\varepsilon \tag{4}$$

depend on the Schmidt numbers for $k(\sigma_k)$ and $\varepsilon(\sigma_\varepsilon)$, and on standard $k - \varepsilon$ model constants that default (Launder and Spalding, 1974) to $(C_{\varepsilon 1}, C_{\varepsilon 2}) = (1.44, 1.92)$. Apart from the source terms $(S_.)$, these equations are standard. The budget equation for ε cannot be derived from the actual equations. It has been modelled (Equation (4)) consistently with k (Equation (3)) and the differential (Equation (6)) of Kolmogorov's relation

$$\varepsilon = C_\mu^{\frac{3}{4}} \frac{k^{\frac{3}{2}}}{l_m}, \tag{5}$$

which predicts that

$$\frac{d\varepsilon}{\varepsilon} = \frac{3}{2}\frac{dk}{k}, \tag{6}$$

when $\frac{dl_m}{l_m} \approx 0$.

2.2. Source terms

Because of a possible reduction due to leaf fluttering (Laadhari et al., 1974), the viscous drag might be lower than expected within the canopy. Therefore, the viscous drag can be assumed negligible compared to form drag. Consequently, S_U (Equation (7)) depends on the form drag coefficient (C_X) for the canopy,

$$S_U = -\frac{C_X}{2}U^2, \tag{7}$$

where C_X depends (Thom, 1971) on the one-sided vegetation surface density a_p (in m^{-1}). We note that a_p partly cancels (Massman, 1997) through a dimensionless canopy drag coefficient (C_d) defined as $C_X = 2a_pC_d$ (Wilson and Shaw, 1977). In addition, vegetation elements break the mean flow motion into wake turbulence with a smaller length scale than the shear-generated turbulence. Therefore the canopy creates a net k loss (Green et al., 1995) despite wake enhancement of the k generation rate. This process is due to the rapid dissipation of wake eddies (Raupach and Shaw, 1982). It could be modelled (Green, 1992) with a source term for k (Equation (8)), being the sum of the wake k production rate ($\propto \frac{C_X}{2}U^3$) with a sink ($\propto \frac{C_X}{2}Uk$) to account for the short-circuiting of the turbulence cascade,

$$S_k = \frac{C_X}{2}\left(\beta_P U^3 - \beta_d U k\right). \tag{8}$$

While β_P ($\in$ [0, 1]) is the fraction of mean airflow kinetic energy lost by drag that is converted into k, the dimensionless coefficient for the turbulence cascade short-circuiting (β_d) has no clear physical basis (Green, 1992). For similar reasons, the different models for S_ε have little physical basis, beyond dimensional arguments. The simplest S_ε model (Equation (9)) is a logical extension of Kolmogorov's relation, which yields

$$S_\varepsilon = C_{\varepsilon 4}\frac{\varepsilon}{k}S_k. \tag{9}$$

An alternative model for S_ε is that proposed by Liu et al. (1996), which states that

$$S_\varepsilon = \frac{C_X}{2}\left(C_{\varepsilon 4}\beta_P\frac{\varepsilon}{k}U^3 - C_{\varepsilon 5}\beta_d U\varepsilon\right). \tag{10}$$

This latter (Equation (10)) model was intended as an improvement of the simpler (Equation (9)) S_ε model that could not fit wind-tunnel data (Liu et al., 1996). Moreover, this latter model (Equation (10)) agrees with the fact that the derivation of $C_{\varepsilon 4} = C_{\varepsilon 5} = 3/2$ from the differential of the Kolmogorov relationship lacks a clear rationale: the constant value 3/2 is valid for the bulk ε variation ($d\varepsilon$), while S_ε is just a part of it. The different values ($C_{\epsilon 4} = 1.5$, $C_{\varepsilon 5} = 0.4$) for the alternative S_ε model (Equation (10)) were justified by mixing length anisotropy. While mixing length anisotropy and its variation in the vertical (Katul and Chang, 1999) might be a limitation for $k - \varepsilon$ models, the closure model (Equation (2)) predicts that the mixing length anisotropy rather influences ν_t. Therefore, this latter S_ε model (Equation (10)) might be a palliative for yet undetermined reasons. Anyway, we will herein retain Equation (10)) as a more general form than Equation (9) for S_ε.

3. Derivation of Analytical Constraints

3.1. Canopy flow variables

Within a dense and homogeneous canopy, far enough from any boundary, turbulence length scale measurements indicate that l_m does not vary significantly (Allen, 1968). With this assumption, the modelled momentum budget equation (Equations (1) and (7)) predicts an exponential decay for U (Perrier, 1967), using a first-order closure model,

$$\nu_t = l_m^2 \left(\frac{dU}{dz} \right). \tag{11}$$

This latter exponentially decaying velocity profile is the solution of

$$\frac{dU}{dz} = \gamma U, \tag{12}$$

where

$$\gamma = \frac{C_X}{2} \left(2\alpha^2 \right)^{-\frac{1}{3}}. \tag{13}$$

This follows from a mixing length model that suits even second-order turbulence modelling purposes (Wilson and Shaw, 1977),

$$l_m = \frac{2\alpha}{C_X} \tag{14}$$

and involves its own dimensionless coefficient (α), which need not depend on vegetation density (Seginer, 1974). We expect that the 1.5-closure model of ν_t

(Equation (2)) collapses to the first-order one (Equation (11)) in such a simplified case where both models should be equally valid. This collapse in formulation allows k to be calculated from

$$k = \frac{\left(\frac{\alpha}{2}\right)^{\frac{2}{3}}}{C_\mu^{\frac{1}{2}}} U^2, \tag{15}$$

while ε (Equation (16)) follows from Kolmogorov's relation (Equation (5)) for high Reynolds number flow,

$$\varepsilon = \frac{C_X}{4} U^3. \tag{16}$$

3.2. Source Model Coefficients

A substitution of the analytical expressions for flow variables (Equations (2), (12)–(16)) within the k modelled budget (Equations (3) and (8)) leads to a relation

$$\beta_d = C_\mu^{\frac{1}{2}} \left(\frac{2}{\alpha}\right)^{\frac{2}{3}} \beta_P + \frac{3}{\sigma_k} \tag{17}$$

that does not explicitly rely on C_X or l_m, and therefore might cancel any dependence on vegetation density for the remaining coefficients. Similarly

$$C_{\varepsilon 5}\beta_d = C_\mu^{\frac{1}{2}} \left(\frac{2}{\alpha}\right)^{\frac{2}{3}} \left(C_{\varepsilon 4}\beta_P - \frac{C_{\varepsilon 2} - C_{\varepsilon 1}}{2}\right) + \frac{6}{\sigma_\varepsilon} \tag{18}$$

derives from the ε modelled budget (Equations (4) and (10)). The above Equations (17) and (18) do not provide enough constraints for the less meaningful coefficients (β_d, $C_{\varepsilon 4}$ and $C_{\varepsilon 5}$) to be calculated from the pair (α, β_P). Anyway, β_P, β_d, $C_{\varepsilon 4}$ and $C_{\varepsilon 5}$ should obey any constraint through

$$\left[C_\mu^{\frac{1}{2}} \left(\frac{2}{\alpha}\right)^{\frac{2}{3}} C_{\varepsilon 5}\right]\beta_P + \left[\frac{3C_{\varepsilon 5}}{\sigma_k}\right] = \left[C_\mu^{\frac{1}{2}} \left(\frac{2}{\alpha}\right)^{\frac{2}{3}} C_{\varepsilon 4}\right]\beta_P + \left[\frac{6}{\sigma_\varepsilon} - C_\mu^{\frac{1}{2}} \left(\frac{2}{\alpha}\right)^{\frac{2}{3}} \frac{C_{\varepsilon 2} - C_{\varepsilon 1}}{2}\right] \tag{19}$$

implied by the previous relations (Equations (17) and (18)). Then, we intend to derive a set of coefficient values with the widest generality. LES require $\beta_P \approx 0.1$ for sub-grid scale turbulence generation (Kanda and Hino, 1994), while published $k-\varepsilon$ models used $\beta_P = 1$. Therefore, Equation (19) must be of a degenerate type in order to be satisfied with a single set of constants for any β_P value.

Otherwise, each set of coefficients (β_P, β_d, $C_{\varepsilon 4}$, $C_{\varepsilon 5}$) for the solution of Equation (19) would be unique, and thus lose any generality. The coefficients of the linear relations with β_P on both sides of (Equation (19)) must then be equal. This implies

$$C_{\varepsilon 4}\,(=C_{\varepsilon 5})=\sigma_k\left(\frac{2}{\sigma_\varepsilon}-\frac{C_\mu^{\frac{1}{2}}}{6}\left(\frac{2}{\alpha}\right)^{\frac{2}{3}}(C_{\varepsilon 2}-C_{\varepsilon 1})\right) \tag{20}$$

and fixes $C_{\varepsilon 4}$ and $C_{\varepsilon 5}$ values independently of (β_P, β_d). This independence is consistent with the modelled decoupling between S_ε (Equation (10)) coefficients. This finally checks that our estimates based on Equations (11)–(13), (15) and (16) are particular model solutions when the mixing length (Equation (14)) is a constant and Kolmogorov's relation (Equation (5)) holds.

4. Conclusions

We derived two equations relating source term model coefficients. The former Equation (17) predicts the coefficient β_d for the turbulence cascade bypass within the canopy as a function of the wake k production coefficient (β_P), the dimensionless product (α) of the mixing length with the drag coefficient, and standard $k-\varepsilon$ model constants. Equation (20) gives the coefficient $C_{\varepsilon 4}$, which relates the source term for dissipation (S_ε) to that for k (S_k). Equation (20) has been designed to suit a broad range of canopy densities and LES sub-grid scale turbulence modelling, through its independence of (β_P, β_d). Equation (20) asserts that $C_{\varepsilon 4}$ is a constant for any given set of standard $k-\varepsilon$ model constants, if α also is a constant. These relations are required, but are not necessarily sufficient for the prediction of a high Reynolds number flow within the canopy. These expressions rely on a particular solution of the 1.5-order closure model within the canopy, and this analytical solution might help us derive similar expressions for various two-equation turbulence models, by analogy. Moreover, the substitution of this solution within modelled budget equations proves that Equation (10) for S_ε induces an implicit dependence between model coefficients. This dependence is not consistent with the reported forms for S_ε. This S_ε model has been shown to improve k predictions, in contrast with the model (Equation (9)). Among the possible explanations for Equation (9) failure, we note that the pair $(\beta_P, \beta_d) = (1, 4)$ of values used to diagnose Equation (9) failure do not obey Equation (17). Moreover, setting $\beta_P = 1$ implies that no mean kinetic energy is lost by viscous drag. This latter assumption seems unrealistic, though the β_P value might be significantly higher than 0.5 in most cases and possibly varying with vegetation density.

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References

Allen, L. H. J.: 1968, 'Turbulence and Wind Spectra within a Japanese Larch Plantation', *J. Appl. Meteorol.* **7**, 73–78.

Claussen, M.: 1988, 'Models of Eddy Viscosity for Numerical Simulation of Horizontally Inhomogeneous Neutral Surface-Layer Flow', *Boundary-Layer Meteorol.* **42**, 337–369.

Detering, H. W. and Etling, D.: 1985, 'Application of the $E-\varepsilon$ Turbulence Model to the Atmospheric Boundary Layer', *Boundary-Layer Meteorol.* **33**, 113–133.

Green, S. R.: 1992, 'Modelling Turbulent Air Flow in a Stand of Widely-Spaced Trees', *Phoenics J.* **5**, 294–312.

Green, S. R., Grace, J., and Hutchings, N. J.: 1995, 'Observations of Turbulent Air Flow in Three Stands of Widely Spaced Sitka Spruce', *Agric. For. Meteorol.* **74**, 205–225.

Kanda, M. and Hino, M.: 1994, 'Organized Structures in Developing Turbulent Flow within and above a Plant Canopy, Using a Large Eddy Simulation', *Boundary-Layer Meteorol.* **68**, 237–257.

Katul, G. G. and Chang, W. H.: 1999, 'Principal Length Scales in Second-Order Closure Models for Canopy Turbulence', *J. Appl. Meteorol.* **38**, 1631–1643.

Laadhari, F. et al.: 1994, 'Turbulence Reduction in a Boundary Layer by a Local Spanwise Oscillating Surface', *Phys. Fluids* **A6**, 3218–3220.

Launder, B. E. and Spalding, D. B.: 1974, 'The Numerical Computation of Turbulent Flows', *Comp. Meth. Appl. Mech. Eng.* **3**, 269–289.

Lee, X. H.: 1996, 'Turbulence Spectra and Eddy Diffusivity over Forests', *J. Appl. Meteorol.* **35**, 1307–1318.

Liu, J., Chen, J. M., Black, T. A., and Novak, M. D.: 1996, '$E - \varepsilon$ Modelling of Turbulent Air-Flow Downwind of a Model Forest Edge', *Boundary-Layer Meteorol.* **77**, 21–44.

Massman, W. J.: 1997, 'An Analytical One-Dimensional Model of Momentum Transfer by Vegetation of Arbitrary Structure', *Boundary-Layer Meteorol.* **83**, 407–421.

Perrier, A.: 1967, 'Approche théorique de la microturbulence et des transferts dans les couverts végétaux en vue de l'analyse de la production végétale', *La Météorologie* **4**, 527–550.

Raupach, M. R. and Shaw, R. H.: 1982, 'Averaging Procedures for Flow within Vegetation Canopies', *Boundary-Layer Meteorol.* **22**, 79–90.

Seginer, I.: 1974, 'Aerodynamic Roughness of Vegetated Surfaces', *Boundary-Layer Meteorol.* **5**, 383–393.

Thom, A. S.: 1971, 'Momentum Absorption by Vegetation', *Quart. J. Roy. Meteorol. Soc.* **97**, 414–428.

Wilson, N. R. and Shaw, R. H.: 1977, 'A Higher Closure Model For Canopy Flow', *J. Appl. Meteorol.* **16**, 1197–1205.